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A DIFFERENT AND MORE DIRECT APPROACH TO THE INVERSE EDGE BACKSCATTERING PROBLEM

APRIL 1968

N. M. Tomljanovich

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Work Performed for

ADVANCED RESEARCH PROJECTS AGENCY

Contract Administered by

DEVELOPMENT ENGINEERING DIVISION
DIRECTORATE OF PLANNING AND TECHNOLOGY
ELECTRONIC SYSTEMS DIVISION
AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
L. G. Hanscom Field, Bedford, Massachusetts

Sponsored by
Advanced Research Projects Agency
Project Defender
ARPA Order No. 596

Project 8051

Prepared by
THE MITRE CORPORATION
Bedford, Massachusetts
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FOREWORD

The work reported in this document was performed by The MITRE Corporation, Bedford, Massachusetts, for Advanced Research Projects Agency; the contract was monitored by the Directorate of Planning and Technology, Electronic Systems Division, of the Air Force Systems Command under Contract AF 19(628)-5165.

REVIEW AND APPROVAL

Publication of this technical report does not constitute Air Force approval of the report's findings or conclusions. It is published only for the exchange and stimulation of ideas.

A. P. TRUNFIO
Project Officer
Development Engineering Division
Directorate of Planning and Technology

ABSTRACT

In a report[1] R. M. Lewis has shown how the body motion and geometric parameters of an edge located on a torque-free axi-symmetric conducting body could be determined by knowing the short pulse backscattered cross sections of the isolated edge at two different polarizations as functions of time.

The purpose of this report is to indicate another more direct and more practical approach to the inverse edge backscattering problem still utilizing the same measured data as Lewis's. The convenience of one approach over the other will become apparent in this paper.

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SECTION I

INTRODUCTION

In a report[1] R. M. Lewis has shown how the body motion and geometric parameters of an edge located on torque-free axisymmetric conducting bodies could, in principle, be determined by knowing the short-pulse backscattered cross sections of the isolated edge at two different polarizations as functions of time, or by measuring the time history of the short pulse polarization scattering matrix.

The required short pulse must be short enough to isolate a particular edge, but not so short that in effect one could not treat the edge scattering as essential narrowband scattering.

Lewis first determines the local direction of the backscattered edge by maximizing the cross section as a function of the polarization angle. The behavior of the edge direction as a function of time for torque-free axisymmetric conducting bodies can then be utilized to determine the kinematic parameters of the edge, which are the same as those of the body.

Once the kinematic parameters of the body are known, the geometric parameters of the edge can be obtained if a large range of aspect angles is available.

Lewis's paper mentions that there are many different ways of determining the useful parameters, and some might be more direct and more practical than the one introduced. It is the purpose of this paper to indicate a more direct and practical approach for

determining the body motion parameters of torque-free axisymmetric conducting bodies and hence the geometric parameters of the observed edge.

The method utilizes the same basic data as Lewis's method, but the theoretical analysis to obtain the body motion parameters is quite different. In Section II, edge backscattering will be introduced in the light of our approach and a direct way of attacking the inverse problem will be considered.

The method of obtaining body motion parameters for general torque-free conducting bodies of revolution will then be developed and finally, Section IV will indicate that, once the kinematic parameters are known, the method for determining geometric parameters is identical to Lewis's. The convenience of one method over the other will become apparent in this paper.

SECTION II

EDGE BACK-SCATTERING

Using geometrical theory of diffraction[2,3], one can calculate[1] the diffraction coefficients for electromagnetic backscattering from an edge to be

$$d_{\substack{1\\2}} = \frac{e^{i\pi/4}\ \sin(\pi/q)}{q(2\pi k)^{1/2}} \left[(\cos\pi/q - 1)^{-1} \mp (\cos\pi/q - \cos\frac{2\theta + \pi}{q})^{-1}\right] \tag{1}$$

where

$$q = 2 - \frac{\gamma}{\pi}\ ,\ k = \frac{2\pi}{\lambda}$$

and where the angles γ and θ and the subscripts "1", "2", referring respectively to directions $||$ and $\perp$ to the local edge direction, are shown in Figure 1.

The total edge backscattering cross section for incident polarization, making an angle ϕ with the edge orientation, is given by

$$\sigma = \frac{4\pi\rho}{2\hat{N}\cdot\hat{I}} \left[|d_1|^2 \cos^2\phi + |d_2|^2 \sin^2\phi\right] \tag{2}$$

where $\hat{I}$ is the unit vector in the incident field direction. $\hat{N}$ is the unit normal vector to the edge, and ρ is the local radius of curvature of the edge at the point which contributes to the scattering. From Equation (2), one can also write:

$$\sigma = b_1{}^2\cos^2\phi + b_2{}^2\sin^2\phi \tag{3}$$

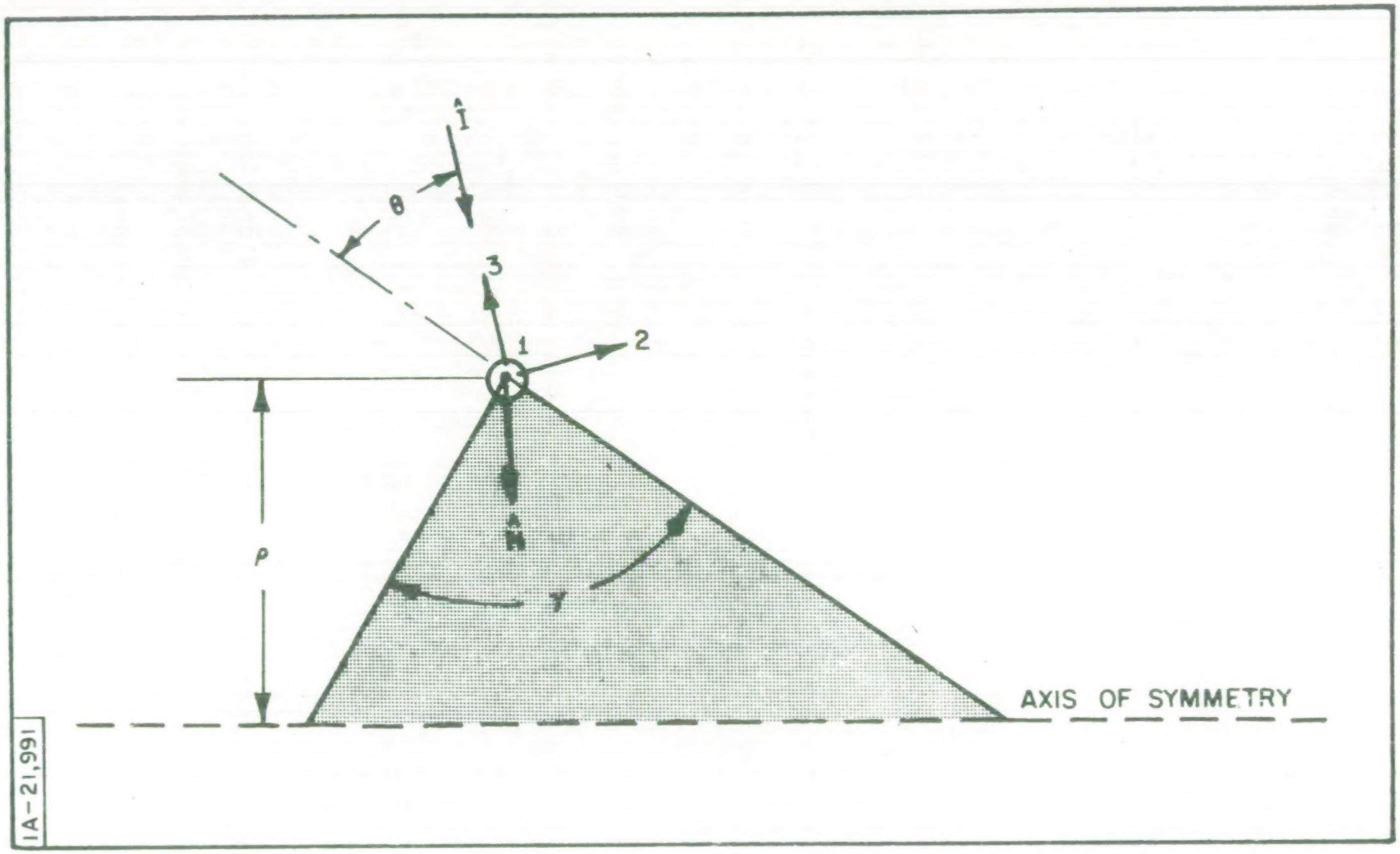

Figure 1. Edge Geometry

where

$$b_{\frac{1}{2}} = \left[\frac{4\pi\rho}{2\hat{N}\cdot\hat{I}} \; |d_{\frac{1}{2}}|^2\right]^{1/2} = \sqrt{\frac{\rho}{\hat{N}\cdot\hat{I}}} \; \frac{\sin(\pi/q)}{q \; k^{1/2}} \left[(\cos\pi/q - 1)^{-1} \mp \right. \tag{4}$$

$$\left. \mp \left(\cos\pi/q - \cos\frac{2\theta + \pi}{q}\right)^{-1}\right]$$

The b_1^2 and b_2^2 represent the measured cross sections of the edge for polarization along and perpendicular to the edge direction; being more directly measurable than the diffraction coefficients, they will be used throughout this paper.

If the edge were a circular edge on a body of revolution, one sees from Equation (4) that b_1 and b_2 contain information about the dynamics of the body in the term $\frac{1}{\sqrt{\hat{N}\cdot\hat{I}}}$ and in the term $\left[(\cos\pi/q - 1)^{-1} \mp \left|\cos\pi/q - \cos\frac{2\theta + \pi}{q}\right|^{-1}\right]$ (because of θ); however, the sum $\left(b_1 + b_2\right)$ which is

$$b_1 + b_2 = \sqrt{\frac{\rho}{\hat{N}\cdot\hat{I}}} \; \frac{\sin(\pi/q)}{q \; k^{1/2}} \; (\cos\pi/q - 1)^{-1} \; , \tag{5}$$

contains information about the dynamics of the body only in the term $\frac{1}{\sqrt{\hat{N}\cdot\hat{I}}}$, simplifying considerably the inverse problem.

Indeed, one can write

$$b_1 + b_2 = g(\gamma,\rho)\frac{1}{\sqrt{\hat{N}\cdot\hat{I}}} \tag{6}$$

where

$$g(\gamma,\rho) = \frac{\sqrt{\rho}\ \sin(\pi/q)}{q \qquad k^{1/2}}\ (\cos \pi/q - 1)^{-1} \tag{7}$$

is a function of the geometrical parameters of the edge and, for a circular edge it is a constant.

Using a short-pulse radar, it is possible to measure the scattering of the isolated point on the edge contributing to the backscattering. As the circular edge on the body is viewed at different directions, the point which contributes to the backscattering moves along the edge and hence the rotation of the unit vector $\hat{N}$ must properly be accounted for.

A mathematical expression for $\hat{N}$ can easily be derived; the unit vector must lie in a plane formed by the axis of symmetry and the line of sight to the particular edge, and thus can be written as:

$$\hat{N} = C_1\hat{I} + C_2\hat{L} \tag{8}$$

where $\hat{L}$ is the unit vector along the axis of symmetry of the body of revolution (Figure 2). The constant C_1 and C_2 can be determined from the following two conditions:

$$\hat{N} \cdot \hat{L} = 0 \qquad\qquad \hat{N} \cdot \hat{N} = 1 \tag{9}$$

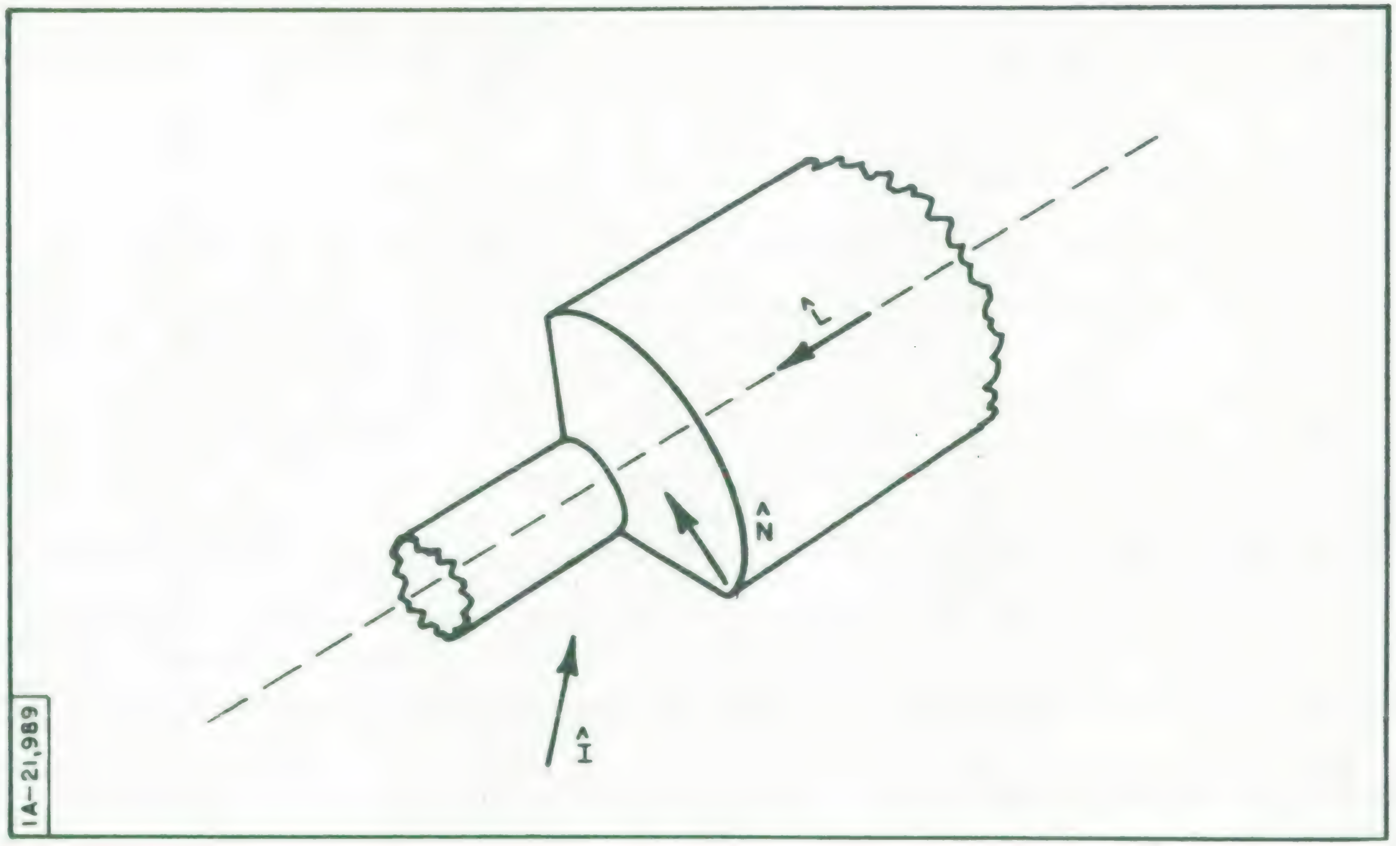

Figure 2. Geometrical Relation Between the Unit Vectors $\hat{I}$, $\hat{L}$, $\hat{N}$

The first condition gives

$$C_1 \hat{I}\cdot\hat{L} + C_2 = 0 \tag{10}$$

or

$$\hat{N} = C_1 [\hat{I} - (\hat{I}\cdot\hat{L}) \hat{L}] \tag{11}$$

and, utilizing the second condition, one obtains

$$C_1 = \pm \frac{1}{\sqrt{1 - (\hat{I}\cdot\hat{L})^2}} \tag{12}$$

therefore

$$\hat{N} = \pm \frac{[\hat{I} - (\hat{I}\cdot\hat{L}) \hat{L}]}{\sqrt{1 - (\hat{I}\cdot\hat{L})^2}} \tag{13}$$

The inspection of Figure 2 sets the sign at plus so that

$$N = + \frac{[\hat{I} - (\hat{I}\cdot\hat{L}) \hat{L}]}{\sqrt{1 - (\hat{I}\cdot\hat{L})^2}} \tag{14}$$

Using the above expression for $\hat{N}$ in Equation (6), one gets

$$b_1 + b_2 = g(\gamma,\rho) \frac{1}{[1 - (\hat{I}\cdot\hat{L})^2]^{1/4}} \tag{15}$$

and

$$\left(b_1 + b_2\right)^2 = g^2(\gamma,\rho)\ \frac{1}{\sqrt{1 - (\hat{I}\cdot\hat{L})^2}} \tag{16}$$

Thus, knowing $(b_1 + b_1)^2$ as functions of time, one would expect that, if the body of revolution were undergoing simple torque-free motion, the above expression should give the body-motion parameters. Before considering a method of obtaining these parameters, let us investigate first how from the measured data one can determine $(b_1 + b_2)^2$ as a function of time, where b_1^2 and b_2^2 are the cross sections for linear polarizations along and perpendicular to the edge orientation.

The measurement procedure, which we establish, is the following: not knowing the edge orientation, the radar performs a two step measurement sequence at each instant of time. First, it transmits a signal which is linearly polarized along a horizontal basis vector and measures the scattering amplitudes S_{11} and S_{21} in the dual linear polarization receivers. Second, it transmits a signal which is linearly polarized along a vertical basis vector and again measures the scattering amplitudes S_{12}, S_{22} in the dual linear polarization receivers.

Thus at each instant of time, the radar measures the polarization scattering matrix [S] for the isolated edge,

$$[S] = \begin{pmatrix} S_{11} & S_{21} \\ S_{12} & S_{22} \end{pmatrix} \tag{17}$$

Assuming the propagation and the target media to be isotropic, the scattering matrix will then be symmetrical, namely

$$S_{12} = S_{21} \ . \tag{18}$$

If the radar had the horizontal polarization basis vector aligned along the edge orientation, then according to the geometrical theory of diffraction[3], the scattering matrix [S'] would be diagonal

$$[S'] = \begin{pmatrix} b_1 & 0 \\ 0 & b_2 \end{pmatrix} \tag{19}$$

where the b's are the quantities which we seek to determine.

Nevertheless, having measured [S], one can always find [S'] and the angle ϕ between the radar horizontal basis vector and the edge direction by solving the classical <u>eigenvalue problem</u>.

In our case, however, since one does not need to know ϕ, and b_1 and b_2 separately, but only their sum $(b_1 + b_2)$, the trace of the diagonal matrix [S'] , the problem is considerably simplified.

The measured scattering matrix [S], being the scattering matrix for the edge direction oriented at an angle ϕ with respect to the horizontal polarization basis vector, can be obtained from [S'] by a $[-\phi]$ rotation of the coordinates of [S'].

In matrix form,the transformation is

$$[S] = [R]_{-\phi}\ [S']\ [R]_{-\phi}^{-1} \tag{20}$$

where $[R]_\phi$ = the rotation matrix = $\begin{pmatrix} \cos\phi & \sin\phi \\ -\sin\phi & \cos\phi \end{pmatrix}$ (21)

Using Equations (19, 20, and 21) one can then obtain that

$$[S] = \begin{pmatrix} \alpha + \beta\cos 2\phi & \beta\sin 2\phi \\ \beta\sin 2\phi & \alpha - \beta\cos 2\phi \end{pmatrix} \tag{22}$$

where $\alpha = \dfrac{b_1 + b_2}{2}$ and $\beta = \dfrac{b_1 - b_2}{2}$ (23)

Obtaining the trace of [S] one notes that

$$S_{11} + S_{22} = b_1 + b_2 \tag{24}$$

The result is not unexpected since it is well known that the trace of a matrix remains invariant under unitary transformation, such as rotation, for instance.

One then obtains $(b_1 + b_2)$ by simply measuring the diagonal elements of the polarization scattering matrix [S] and adding them.

In the next section we will consider how, knowing $(b_1 + b_2)$ as function of time, it is possible to determine the body motion parameters of the edge.

SECTION III

DETERMINATION OF BODY MOTION PARAMETERS

As previously stated, the time history of the known quantity $(b_1 + b_2)^2$ contains information about the dynamics of any isolated edge located on a torque-free axisymmetric object. From Equation (16), which is

$$(b_1 + b_2)^2 = g^2(\gamma,\rho) \frac{1}{\sqrt{1 - (\hat{I}\cdot\hat{L})^2}} \tag{16}$$

where

$$g(\gamma,\rho) = \frac{\sqrt{\rho} \sin \pi/q}{q \quad k^{1/2}} (\cos \pi/q - 1)^{-1} \tag{17}$$

one sees that the term $\hat{I}\cdot\hat{L}$ must next be expressed in terms of the body motion parameters of the torque-free body.

This was done for general torque-free axisymmetric targets, by the author in another report; from Figure 3, for the case that $R \gg$ (size of the target), the unit vectors $\hat{I}$ and $\hat{L}$ are

$$\hat{I} = \frac{X}{R} \hat{i} + \frac{Z}{R} \hat{k} \tag{30}$$

$$\hat{L} = \sin \theta' \cos(\omega t + \psi) \hat{i} + \sin \theta' \sin(\omega t + \psi) \hat{j} + \cos \theta' \hat{k}$$

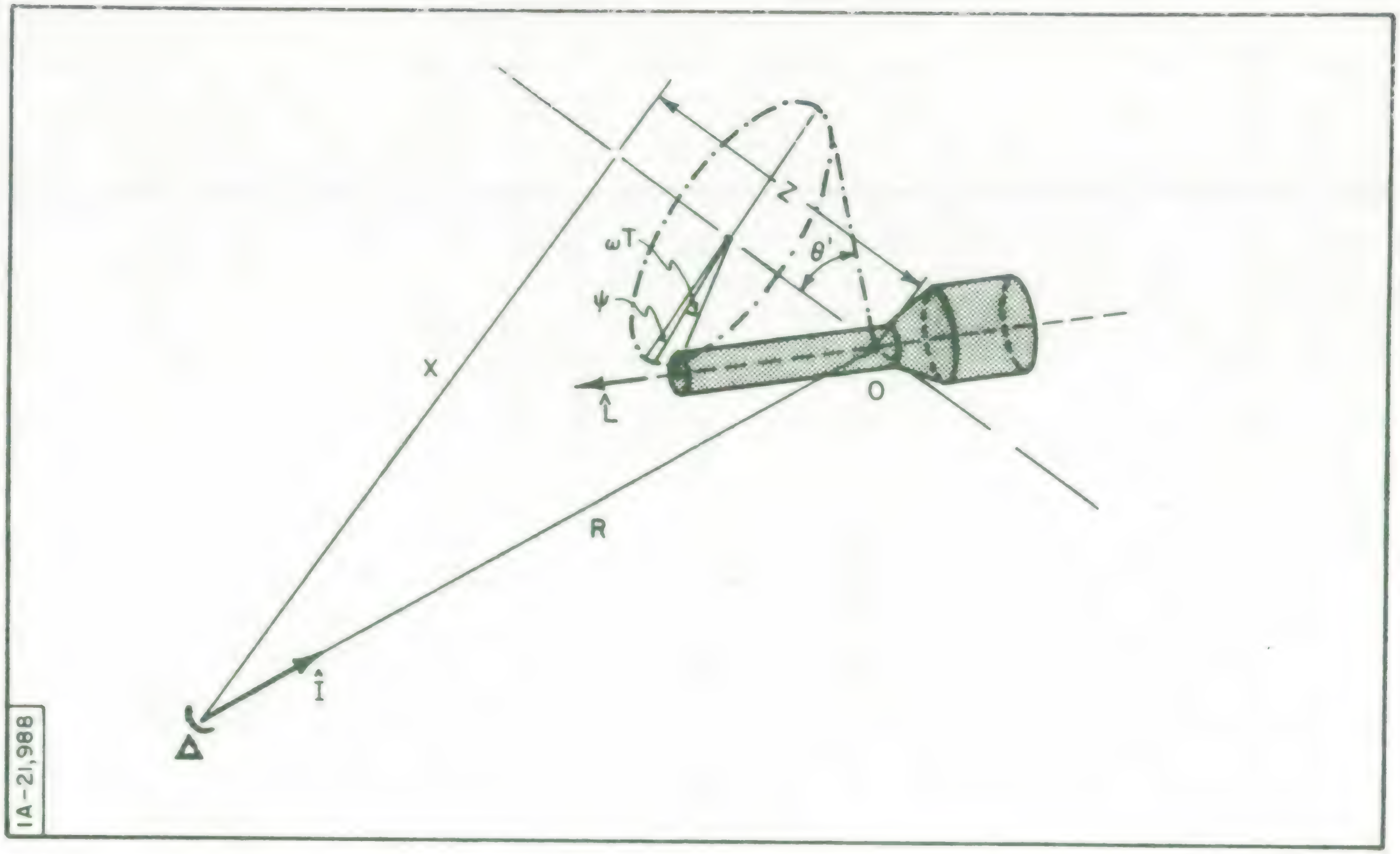

Figure 3. Geometry of a General Torque-Free Axisymmetric Target

where R is the range of the object, Z and X are the projections of R along the instantaneous axis of precession of the object and $\perp$ to it, θ' is the semi-angle of precession, ω is the precessional frequency, and ψ is the phase angle indicating the initial position of the precessing motion relative to the radar frame.

Then $\hat{I} \cdot \hat{L} =$

$$\frac{1}{R}\left[Z \cos \theta' + X \sin \theta' \cos(\omega t + \psi)\right]$$

Thus, one clearly sees that all body motion parameters are included in the quantity $(b_1 + b_2)^2$ vs. time, and the extraction of these parameters is our next task.

There are several ways of proceeding:

a) One way of obtaining the kinematic parameters is to consider the quantity

$$S = \frac{1}{(b_1 + b_2)^4}$$

which in terms of $\hat{I} \cdot \hat{L}$ from Equation (15) is:

$$S = \frac{1}{(b_1 + b_2)^4} = \frac{1 - (\hat{I} \cdot \hat{L})^2}{g^4(\gamma, \rho)} \tag{31}$$

Since $$(\hat{I}\cdot\hat{L})^2 = \frac{1}{R^2}\left[Z\cos\theta' + X\sin\theta'\cos(\omega t + \psi)\right]^2$$

$$= \frac{1}{R^2}\left[Z^2\cos^2\theta' + X^2\sin^2\theta'\cos^2(\omega t + \psi) + ZX\sin 2\theta'\cos(\omega t + \psi)\right]$$

$$= \frac{1}{R^2}\left\{\left(Z^2\cos^2\theta' + \frac{X^2\sin^2\theta'}{2}\right) + ZX\sin 2\theta'\cos(\omega t + \psi) + \frac{X^2\sin^2\theta'\cos 2(\omega t + \psi)}{2}\right\} \tag{32}$$

The known quantity S is then:

$$S = \frac{1}{g^4(\gamma,\rho)}\left\{\left(1 - \frac{Z^2\cos^2\theta'}{R^2} - \frac{X^2\sin^2\theta'}{2R^2}\right) - \frac{ZX\sin 2\theta'}{R^2}\cos(\omega t + \psi) - \frac{X^2\sin^2\theta'}{2R^2}\cos 2(\omega t + \psi)\right\} \tag{33}$$

A harmonic analysis will give the precessional frequency and the phase angle ψ. Furthermore, the amplitudes of the constant term A_o, of the 1st harmonic A_1 and the 2nd harmonic A_2 , which are respectively

$$A_o = \frac{1}{g^4(\gamma,\rho)}\left(1 - \frac{Z^2\cos^2\theta'}{R^2} - \frac{X^2\sin^2\theta'}{2R^2}\right) \tag{34}$$

$$A_1 = \frac{1}{g^4(\gamma,\rho)} \left(\frac{ZX\sin 2\theta'}{R^2} \right) \tag{35}$$

$$A_2 = \frac{1}{g^4(\gamma,\rho)} \left(\frac{X^2\sin^2\theta'}{2R^2} \right) \tag{36}$$

together with the equation

$$R^2 = Z^2 + X^2 \tag{37}$$

could be used to solve for the four unknowns:

$$g^4(\gamma,\rho),\ Z,\ X,\ \theta' .$$

The above approach, in contrast to Lewis's, does not require a large range of measurements, making it very suitable for rapid determination of the dynamics of unstabilized torque-free objects; however, because of the $\frac{1}{(b_1+b_2)^4}$ term, it could be exceedingly sensitive to errors, requiring very exact measurements of $(b_1 + b_2)$.

b) Another method of extracting the body motion parameters from Eq. 16 and which might not be as sensitive to errors as the previous approach is the following:

Since in most cases

$$(\hat{I}\cdot\hat{L}) < 1 ,$$

one could use the binomial expansion to express the quantity $(b_1+b_2)^2$ as

$$(b_1+b_2)^2 = \frac{g^2(\gamma,\rho)}{\sqrt{1-(\hat{I}.\hat{L})^2}} = g^2(\gamma,\rho)\left[1 + \tfrac{1}{2}(\hat{I}.\hat{L})^2 + \frac{3}{8}(\hat{I}.\hat{L})^4 + \frac{15}{36}(\hat{I}.\hat{L})^6 + \ldots\right]$$

for the case that $\hat{I}.\hat{L}$ is small or the case when the body's axis of precession is almost $\perp$ to the line of sight and for a sufficiently small angle of precession, one can assume that $(\hat{I}.\hat{L})^4$ and higher order terms can be neglected, so that

$$(b_1 + b_2)^2 \simeq g^2(\gamma,\rho) \left|1 + \frac{1}{2}(\hat{I}.\hat{L})^2\right|$$

$$= g^2(\gamma,\rho) \left|1 + \frac{1}{2R^2}\left(Z^2\cos^2\theta' + \frac{X^2\sin^2\theta'}{2}\right) + \frac{ZX\sin 2\theta'}{2R^2}\cos(\omega t + \psi)\right.$$

$$\left. + \frac{X^2\sin^2\theta'}{4R^2}\cos 2(\omega t + \psi)\right| \qquad (39)$$

By performing a harmonic analysis of $(b_1 + b_2)^2$ versus time the precessional frequency ω and the phase angle of precession could be obtained. Again, by knowing the amplitude for the constant part A_o, the 1st harmonic A_1, and the second harmonic A_2, one could solve the set of equations:

$$A_o = g^2(\gamma,\rho) \left[1 + \frac{1}{2R^2} \left(Z^2\cos^2\theta' + \frac{X^2\sin^2\theta'}{2}\right)\right] \tag{40}$$

$$A_1 = g^2(\gamma,\rho) \frac{ZX\sin 2\theta'}{2R^2} \tag{41}$$

$$A_2 = g^2(\gamma,\rho) \frac{X^2\sin^2\theta'}{4R^2} \tag{42}$$

$$R^2 = X^2 + Z^2 \tag{43}$$

to determine the remaining unknown body motion parameters.

This latter approach, although not as sensitive to errors as the first approach, requires a more specific range of aspect angles, being only applicable in the vicinity of the region where $\hat{I}.\hat{L} = 0$ or where $(b_1 + b_2)^2$ vs. time is a simple periodic function. This again limits the usefulness of the approach, when trying to rapidly determine the dynamics of torque-free bodies.

SECTION IV

CONCLUSION

This paper has illustrated a way of obtaining body motion parameters different from that indicated by Lewis[1]; however, once the body motion parameters are obtained, the method of determining the geometrical parameters of the edge are identical to Lewis's[1]. In short, knowing the dynamics of the edge, one utilizes fully the time history of each measured cross section b_1 and b_2 to extract the geometrical parameters: ρ, γ, θ . Indeed, having 3 unknowns, it is sufficient to measure one of the b's at 3 different times to obtain a set of equations whose solution should give the 3 unknowns. It is very clear that both Lewis's method and the approach described in this paper are very useful for determining the kinematic and geometric parameters of torque-free axisymmetric targets; furthermore, having different approaches, the radar operator has more leeway to choose the one which, for the particular case, is more practical and less sensitive to errors.

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In a report[1] R. M. Lewis has shown how the body motion and geometric parameters of an edge located on a torque-free axi-symmetric conducting body could be determined by knowing the short pulse backscattered cross sections of the isolated edge at two different polarizations as functions of time.

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